AUTOREGRESSIVE ANALYSIS OF VARIANCE FOR EXPERIMENTS WITH SPATIAL DEPENDENCE BETWEEN PLOTS: A SIMULATION STUDY

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- ABSTRACT: The analysis of variance remains one of the most appreciated techniques of field experiment, even despite almost a hundred years of its first proposal. However, in many cases, its application can be several impaired due the fact of lack – or even forgotten - of assumptions. In several experiments, the researchers make use of blocks to control the local heterogeneity, nevertheless, in some cases, only this it cannot be enough, especially in experiments where the data have some kind of spatial dependence. Therefore, to increase the accuracy of comparisons between treatments, an alternative is to consider the study of the spatial dependence of the variables in the analysis. With the knowledge of the relative positions of the plots (referenced data), the spatial variability can be used as a positive factor, collaborating with the experimental results. To develop this study we used data generated by simulation. The data was generated according a Randomized Complete Block Design (RCBD), with eighteen and five treatments per block; and several scenarios of spatial dependence in the error. We compared the non-spatial analysis (which considers the errors independent) with spatial analysis (analysis of variance considering the autoregressive model - ANOVA-AR). The use of spatial statistical tools in the analysis of data increased the precision of the analysis, through the reduction of the Mean Squared Error. We also noticed a reduction of Mean Squared Block and Mean Squared Treatment. The greater reduction was notice in ANOVA-AR3 for great part of the simulated scenarios, mainly in those with strong spatial dependence. The experiments with a small number of treatments per block did not present a reduction of Mean Squared Error, however, the reduction of Mean Squared Block and Mean Squared Treatment, ally to the fact that data are spatial dependent justify the use of ANOVA-AR.

- KEYWORDS: Autoregressive model, geostatistics, ANOVA-AR.

1 Introduction

The biggest challenge when conducting an experiment is to compare treatments with the greatest possible accuracy, to have security in the inferences to be made from the results. The accuracy of an experiment is directly connected to small changes in experimental units. These small changes can cause heterogeneity between plots, also known as random variation, environmental variation or simply experimental error (STORCK, 2000).

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When the experimental error is too large, the statistical tests used to compare means of treatments may be influenced and some differences between treatment means can be undetected. Keeping this in view, the agricultural experimentation increasingly requires the use of more refined experimental techniques and data analysis. In this refinement are several factors involved, such as careful choice of design, installation locations of the experiments, number of repetitions, size and shape of the parcel and a perfect conducting experiment (BANZATTO e KRONKA, 2006). All of these factors influence directly or indirectly the experimental precision.

One of the most used design are the Randomized Complete Block Design (RCDB). When the nuisance source of variability is known and controllable, a design technique called blocking can be used to systematically eliminate its effect on the statistical comparisons among treatments (MONTGOMERY, 2001).

Nevertheless, in several experiments, this source of variability cannot be adequately controlled. This becomes clearer in experiments with some kind of spatial correlation, in which classical ANOVA could be ineffective or inaccurate. Gumpertz et al. (1997) suggest that for some experiments with randomized block design, analysis of variance that takes into account the spatial analysis can achieve a substantial gain in accuracy. Katsileros et al. (2015) said that whenever variability cannot be controlled by blocking, nearest neighbour methods could be helpful.

With the computational development, some alternatives have become available to researchers, for example, the spatial analysis methods of experiments. The literature lists numerous methods proposed and adopted to model the spatial variation, such as fitting the nearest neighbour (BARTLETT, 1978; PAPADAKIS, 1937; WILKINSON et al., 1983; ZIMMERMAN; HARVILLE, 1991), least squares smoothing (GREEN et al., 1985; SMITH and CASLER, 2004; YANG et al., 2004), kriging (BRESLER et al., 1981; LOPEZ and ARRUE, 1995) and more recently, the use of spatial P-splines (RODRIGUEZ-ÁLVAREZ et al., 2018; VELAZCO et al., 2017).

Of these methods, modelling the spatial structure through a separable autoregressive (AR1 ⊗ AR1) has become attractive and is commonly used in agricultural experimentation (BRAYSHER et al., 2001; CULLIS and GLEESON, 1991; GILMOUR et al., 1997; SCOLFORO et al., 2016; SINGH et al., 2003; YANG et al., 2004).

Long (1996) describes an autoregressive analysis of variance that considers the proximity pattern (GUMPERTZ et al., 1997). This methodology takes into consideration not only the nearest neighbour, or the autocorrelation in row and column (AR1 ⊗ AR1), but neighbours who are within a certain radius of distance.

Considering the methodology proposed by Long (1996), this study analysed (by simulation) 15,000 datasets generated according a RCBD. All the datasets have spatial dependence in the error following a nonlinear Gaussian Spatial model of correlation, with several levels of spatial dependence. The aim was to determine whether the autoregressive analysis of variance (ANOVA-AR) has higher accuracy than the classical analysis of variance when faced with spatially dependent data. These was done by comparison of Mean Squared Error, Mean Squared Block and Mean Squared Treatment and though the construction of Monte Carlo Confidence Intervals.
2 Material and methods

The autoregressive analysis of variance (ANOVA-AR) was described by Long (1996). The basic idea consists of transform autocorrelated observations in uncorrelated observations. To do so, we begin with the choice of the proximity pattern defined by Gumpertz et al. (1997). More than one proximity pattern can be adopted. The proximity patterns of the first, second and third order will be represented by AR1, AR2, and AR3, respectively (Figure 1).

![Proximity patterns](image)

Figure 1 - Proximity patterns, where ◆ represents a reference parcel, ● represents the neighboring plots considered in the comparison and □ represents the remaining plots.

To study the spatial pattern of the experiment, we adopt the model SAR (spatial autoregressive), which, according to Griffith (1988) can be defined by:

\[ Y = \rho W Y + X \beta + \varepsilon \quad (1) \]

where \( Y \) is a vector \( n \times 1 \) of observed values, \( \rho \) is a spatial autoregressive parameter, \( W \) is a matrix \( n \times n \) with weight assignments neighborhood space, \( X \) is an incidence matrix \( n \times p \) of fixed effects, \( \beta \) is a vector \( p \times 1 \) of parameters, \( \varepsilon \) is a vector \( n \times 1 \) of errors inherent to each observation.

The matrix \( W \) is obtained by multiplying two other matrices \( D \) and \( C \) (\( W = D \times C \)). The matrix \( C \) of dimension \( n \times n \) is binary and describes the neighborhood of the plots. To obtain the matrix \( C \), first is necessary to define which proximity pattern will be choose. After that, each cell of the matrix will be completed by 0 or 1, indicating if the cell (plot) is neighbor or no of the adjacent cell.

The matrix \( D \) is a diagonal matrix with elements \( 1/k_i \), where \( k_i \) is the sum of the values of matrix \( C \) rows, which allows the matrix \( W \) to have the sum of each line equal to 1. In other words, \( \rho W Y \) are the observed values weighted by the neighborhood and spatial correlation.
One of the forms to estimate the parameter $\rho$ of the SAR model is the method of maximum likelihood (ML). The original solution of the ML estimation of a spatial autoregressive model, originally proposed by Ord (1975), consists in exploring the decomposition of the Jacobian $|I - \rho W|$ in terms of the eigenvalues $\omega_i$ (with $i = 1, 2, \ldots, N$) of the matrix $W$, given by $|I - \rho W| = \prod_{i=1}^{N}(1 - \rho \omega_i)$ (GRIFFITH;1988, 2005).

With the estimated $\hat{\rho}$, we fit the autocorrelated observations in uncorrelated observations through $Y_{adj} = Y - (\hat{\rho} WY - \hat{\rho} \beta_0)$, where $\beta_0$ is the average of the observations.

Obtained the vector $Y_{adj}$ (experimental observations weighted by neighborhood) we proceed with the construction of the ANOVA-AR. The procedure for their construction is similar to the classical ANOVA.

Through analysis of the Mean Square Error (MSE) and Mean Square Parameter (MSP) versus $MSE_{adj}$ and $MSP_{adj}$ (Mean Square Error and Mean Square Parameter for vector $Y_{adj}$) we check if the autoregressive approach contributed to the decrease of the experiment variability, that is, the goal is to verify that the values of $MSE$ and $MSP$ are smaller than $MSE_{adj}$ and $MSP_{adj}$.

To implement the study, we used simulated datasets in two different configurations:

- experiment i - a randomized complete block design (RCBD) with 18 treatments and 6 blocks;
- experiment ii - a randomized complete block design (RCBD) with 5 treatments and 4 blocks.

The two configurations were proposed to verify if the methodologies present the same results in experiments with a large number of treatments in each block and experiments with a small number of treatments in each block. The experiments were constructed considering a regular grid with plot size defined in 1 $\text{u.m}^2$.

The fixed effects of treatment and block were chosen arbitrarily, without loss of generality, since the aim is not to verify whether or not treatment effect or block, but if the ANOVA-AR can increase the accuracy of the experiment.

The statistical model for both situations is given by $y_{ij} = \mu + t_i + b_j + e_{ij}$, where $y_{ij}$ is the observation of the $i$th treatment in the $j$th repetition, being $i = 1, 2, \ldots, a$ and $j = 1, 2, \ldots, b$, with $a = 18$ and $b = 6$ in the configuration (i) and $a = 5$ and $b = 4$ in the configuration (ii); $\mu$ is an inherent constant to each observation; $t_i$ it is the effect of the $i$th treatment; $b_j$ it is the effect of the $j$th block; $e_{ij}$ it is the experimental error associated to each observation.

To validate the analysis of variance, the error must have mean zero and constant variance, so without loss of generality, the random errors were generated following a normal distribution with mean zero and variance 1. To add the spatial correlation in the error, a Geostatistical model was used. The covariance generated by the Gaussian nonlinear model (equation 2), through transformation $C(h) = C(0) - \gamma(h)$, was add to the error. So, the error became $N(0, \Sigma)$. 

According to Clark (1979) field experimental errors with spatial dependence tend to be Gaussian. Thus, we generated several configurations of dependent errors (choosing different range \( a \), sill \( C_0 + C_1 \) and nugget effect \( C_0 \)).

\[
\gamma(h) = \begin{cases} 
0 & |h| = 0 \\
C_0 + C_1 \left\{1 - \exp\left(-\frac{|h|}{a}\right)\right\}^2 & |h| \neq 0
\end{cases}
\]  

(2)

The range is the distance that the samples are spatially correlated. The ratio between the nugget effect and the sill defines the level of spatial dependence of the data. The dependence can be characterized as strong, moderate or weak (CRESSIE and HARTFIELD, 1966; SEIDEL et al., 2016; SEIDEL and OLIVEIRA, 2014). More information of these geostatistical parameters can be found in Cressie (1993), Isaaks and Srivastava (1989), Govaerts (1997), Chilès and Delfiner (1999) and Journel (1978).

The purpose of working with different configurations of dependent error was to verify the efficacy of autoregressive ANOVA when confronted with several levels of spatial dependence.

Therefore, different ranges were generated for the experiment (i) and different degrees of spatial dependence. Experiment (ii) was constructed similarly. For better interpretation, we use the expression Gaus (nugget effect - sill - range). For example, Gaus (0-1-6) means that the error follows a Gaussian model with zero nugget effect, sill one and range six. All configurations simulated are shown in Table 1.

We simulated 15,000 datasets (through Monte Carlo simulation), a total of 1000 simulations for each configuration error (Table 1). For each dataset we apply the classical ANOVA and the ANOVA-AR with proximity patterns of first, second and third order.

Table 1 – Simulation configuration

<table>
<thead>
<tr>
<th>Experiment i</th>
<th>Experiment ii</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Nugget Effect</td>
</tr>
<tr>
<td>Gaus(0-1-6)</td>
<td>0</td>
</tr>
<tr>
<td>Gaus(0.75-1-6)</td>
<td>0.75</td>
</tr>
<tr>
<td>Gaus(0.25-1-6)</td>
<td>0.25</td>
</tr>
<tr>
<td>Gaus(0-1-4)</td>
<td>0</td>
</tr>
<tr>
<td>Gaus(0.75-1-4)</td>
<td>0.75</td>
</tr>
<tr>
<td>Gaus(0.25-1-4)</td>
<td>0.25</td>
</tr>
<tr>
<td>Gaus(0-1-2)</td>
<td>0</td>
</tr>
<tr>
<td>Gaus(0.75-1-2)</td>
<td>0.75</td>
</tr>
<tr>
<td>Gaus(0.25-1-2)</td>
<td>0.25</td>
</tr>
</tbody>
</table>

For the comparison of results between the classical approach and the autoregressive approach we build a 95% Monte Carlo Confidence Interval (BUCKLAND, 1984) for the Mean Squared Block (MSB), Mean Squared Block Adjusted(\( MSB_{\text{adj}} \)), Mean Squared Error
(MSE), Mean Squared Error Adjusted (MSE$_{adj}$), Mean Squared Treatment (MST) and Mean Squared Treatment Adjusted (MST$_{adj}$).

All analysis were performed in R software (R CORE TEAM, 2018)

3 Results and discussion

Initially, we would like to show a practical example of the performance of ANOVA-AR. Let us consider one simulation of the experiment $i$ with a range of 6 (Gaus-0-1-6; data with strong spatial dependence). Table 2 shows a classical ANOVA. Using ANOVA-AR methodology, and considering a first order proximity pattern, we obtained Table 3.

When we compare a classical ANOVA (Table 2) with ANOVA-AR (Table 3), we notice the reduction of SST. There are a decrease of 56% of MSE, which implies in more precision of ANOVA-AR in comparison with classical ANOVA. In addition, we have a decrease of MSB, showing a substantial gain in accuracy.

In the other hand, we have an increase of MST, however, this, ally with decrease of MSE, produces a bigger value of $F_{Trat}$, that can indicates a more powerful $F$ test to detect the true differences between treatments.

Another point that needs an observation is that not all discussion above, between classical ANOVA and ANOVA-AR, took at point, that if we use the classical ANOVA with spatial dependent data, we are ignoring the assumptions of the analysis of variance.

Table 2 – Example of ANOVA for one simulation of experiment $i$ (Gaus-0-1-6)

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>$F_0$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block</td>
<td>5</td>
<td>292.41</td>
<td>58.48</td>
<td>37.28</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Treatment</td>
<td>17</td>
<td>688.11</td>
<td>40.77</td>
<td>25.80</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Error</td>
<td>85</td>
<td>133.31</td>
<td>1.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>107</td>
<td>1113.83</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3 - Example of ANOVA-AR1 for one simulation of experiment $i$ (Gaus-0-1-6)

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>$F_0$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block</td>
<td>5</td>
<td>35.07</td>
<td>7.01</td>
<td>10.18</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Treatment</td>
<td>17</td>
<td>858.79</td>
<td>50.16</td>
<td>72.84</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Error</td>
<td>85</td>
<td>58.53</td>
<td>0.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>107</td>
<td>952.39</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

After this brief exposure of a single simulation, our focus will be the complete simulation scenario. So, let us consider the experiment $i$ with a range of 6 - Gaus-0-1-6 (data with strong spatial dependence). Analysing the parameter MSE$^3$ (Figure 2), we note that the confidence interval showed a lower magnitude for the ANOVAs-AR. The intervals are not symmetrical, since the concentration of 50% of the observations, defined as the

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$^3$ Just for simplify the text and the discussion, QME and QME$_{Adj}$ (and all similars), will be treated just by QME.

median, is located in a small initial portion of each interval. Because there is overlap between the CIs, statistically, we can't say that they are unequal, however, as they were built from 1000 independent data sets, the interval suggest a reduction of the MSE as the increases of proximity pattern (AR1, AR2, AR3), resulting in increased accuracy of the experiment.

When the MSE was confronted one-by-one between classical ANOVA and ANOVA-AR (AR1, AR2, AR3), the reduction occurs in 96%, 98% and 100% of the experiments. Scolforo et al. (2016) noted the same reduction in a study of tree modeling as strategies for the management of *Eremanthus erythropappus*. The ANOVA-AR showed a reduction of MSE and it was capable to detect differences between treatments, which was unable to detect by classical ANOVA.

For the parameter MST (Figure 2), we found intervals that are more symmetrical. A considerable difference between the intervals of ANOVA-AR3 for others can be verified indicating a possible reduction in the variability of the MST.

The great advantage of the approach autoregressive is noted by examining the parameter MSB (Figure 2). The magnitude of the unsymmetrical intervals reduces mildly until ANOVA-AR2, however, a considerable reduction is seen in the ANOVA-AR3. This result corroborates Yang *et al* (2004) which asserts that the RCBD has a low efficiency in experiments containing a large number of treatments, showing that the analysis of variance autoregressive is able to around this problem by reducing the variability of the block factor. Yang and Juskiw (2011) say that in RCBD, proper blocking can reduce error by maximizing the difference between blocks and maintaining the plot-to-plot homogeneity within blocks,
but blocking is ineffective if heterogeneity between plots does not follow a definite pattern (e.g., spotty soil heterogeneity; unpredictable pest incidence after blocking). In addition, when block size is large (>8-12 plots per block), intra-block heterogeneity is inevitable. Thus, the efficiency of the RCBD is often poor in agronomy trials involving a large number of treatments.

Scolforo et al (2016) showed a reduction of 90% of MSB, when compare classical ANOVA and ANOVA-AR.

Using a Geostatistical approach, Nogueira et al (2013, 2015) also concluded that the use of spatial analysis in field experiments with spatial dependence causes the reduction of the MSB.

For data with moderate spatial dependence (Gaus-0.25-1-6) the behavior is similar to the intervals obtained in Gaus-0-1-6. The parameter MSE presents a similar amplitude for ANOVA and ANOVA-ARs, with a little reduction of median with the increase of proximity pattern. The parameters MSB and MST showed a similar behavior of the configuration Gaus-0-1-6. The data with weak spatial dependence (Gaus-0.75-1-6) exhibit behavior similar to Gaus-0-1-6 and Gaus-0.25-1-6, for parameters MSE and MST. In this configuration, however, we note the largest decrease in the variability of the parameter MSB, showing that the method was more sensitive to this configuration. The variability of the parameter MSB is low compared with Gaus-0-1-6 (intervals between 4.69 and 94.6) and Gaus-0.25-1-6 (intervals between 6.94 and 77.97), because of presents intervals between 8.09 and 65.05. The parameter MSB of the ANOVA-AR3 is not statistically equal to the others, because there are not overlapping intervals.

The experiment i with a range of 4 u.m (Figure 3) and 2 u.m (Figure 4) have similar behavior to experiment i with a range 6 u.m (Figure 2).
Figure 3 - Confidence intervals for experiment $i$ with range 4.

Figure 4 - Confidence intervals for experiment $i$ with range 2.
For the experiment ii (Figure 5 and 6), we notice a little increase of the MSE median. Initially this information indicate a loss of precision. However, it is important to note that the experiment ii has only 20 observations. The spatial analysis, including the Geostatistics tools presents some difficulties to deal with a small number of observations.

Another point that must be considered is that even the ANOVA presenting a lower value of MSE in comparison with ANOVA-AR, it contravenes the assumptions of independent errors. Nogueira et al. (2013, 2015) using a Geostatistical approach of ANOVA obtained an increase of the MSE, that was justified with the increase of the power of F test, in that situation.

The behavior of MST and MSB follow the same as presented in experiment i. We note a reduction of the median with the increase of proximity pattern. Also, we see a smaller confidence intervals amplitude, especially for the parameter MSB.

The ANOVA-AR2 and ANOVA-AR3 weren't apply for this experiment (only 20 observations), because the proximity pattern of 2nd and 3rd order would cause an overlapping of the neighbors, that could mask the real effects of spatial dependence.

![Confidence intervals for experiment ii with range 1.](image)
Conclusions

For the experiment i, the autoregressive approach showed a reduction in variability of block and treatment factors. The decrease in MSE indicates that the autoregressive approach was more precise than the classical approach.

For the experiment ii, the autoregressive approach did not show a reduction of MSE, however, the reduction of MSB and MST, ally to the fact that data are spatial dependent, justify the use of ANOVA-AR, instead classical ANOVA.

Acknowledgements

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RESUMO: A análise de variância continua sendo uma das técnicas mais apreciadas na experimentação de campo, mesmo após quase cem anos de sua primeira proposta. No entanto, em muitos casos, sua aplicação pode ser prejudicada devido à falta - ou mesmo do esquecimento – dos pressupostos. Em vários experimentos, os pesquisadores fazem uso de blocos para controlar a heterogeneidade local, no entanto, em alguns casos, apenas isso pode ser insuficiente,
especialmente em experimentos onde os dados possuem algum tipo de dependência espacial. Assim, para aumentar a precisão das comparações entre tratamentos, uma alternativa é considerar na análise o estudo da dependência espacial das variáveis. Com o conhecimento das posições relativas das parcelas (dados referenciados), a variabilidade espacial pode ser utilizada como um fator positivo, colaborando com os resultados experimentais. Para desenvolver este estudo, foram usados dados gerados por simulação. Os dados foram gerados segundo um delineamento de blocos casualizados (DBC), com dezoito e cinco tratamentos por bloco; e vários cenários de dependência espacial no erro. Compararam a análise não espacial (que considera os erros independentes) com a análise espacial (análise de variância considerando o modelo autoregressivo - ANOVA-AR). O uso de ferramentas estatísticas espaciais na análise de dados aumentou a precisão da análise, através da redução do Quadrado Médio do Erro. Observamos também uma redução do Quadrado Médio do Bloco e do Quadrado Médio do Tratamento. A maior redução foi observada na ANOVA-AR3 na maior parte dos cenários simulados, principalmente naqueles com forte dependência espacial. Os experimentos com um pequeno número de tratamentos por bloco não apresentaram redução do Quadrado Médio do Erro, no entanto, a redução do Quadrado Médio do Bloco e do Quadrado Médio do Tratamento, aliado ao fato dos dados possuírem dependência espacial, justificaram o uso da ANOVA-AR.

- PALAVRAS-CHAVE: Modelo autoregressivo, geoestatística, ANOVA-AR

References


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